

# Quiz 5, Business Calculus

Summer Session I, 2012

9:28

9:33

5

⇒ Give 20 minutes

Name: \_\_\_\_\_

Key

1. (2 points) Find the absolute maximum and minimum (if any) of  $f(x) = -x^2 + 4x + 6$  on the interval  $[0,5]$ .

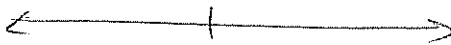
$$f'(x) = -2x + 4$$

$$0 = -2x + 4$$
$$\Rightarrow \boxed{x = 2}$$

$$f(2) = 10$$

$$f(0) = 6$$

$$f(5) = 1$$



⇒ Abs. max of ~~10~~ 10 at  $x = 2$   
Abs. min of 1 at  $x = 5$

2. (2 points) Use logarithm rules to expand and simplify the expression  $\ln \frac{e^x}{x^2}$ .

$$= \ln e^x - \ln x^2$$

$$= x(\ln e) - 2 \ln x$$

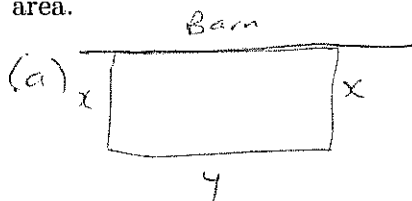
$$= \boxed{x - 2 \ln x}$$

b/c  $\ln e = 1$

3. (4 points) Farmer Bob wants to enclose a rectangular area next to his barn with fencing. One side of the rectangular area will be the side of the barn (which will need no fencing), and the other three sides will be fencing. If Bob has 80 feet of fencing and the width of the rectangular area is  $x$ , do the following:

(a) Find an expression for the area of the rectangle in terms of  $x$ .

(b) Use calculus to find the dimensions of the rectangular area which will give the *maximum* area.



$$2x + y = 80 \Rightarrow y = 80 - 2x$$

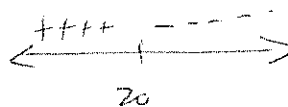
Area =  $x y = x(80 - 2x)$

(a)  $A(x) = 80x - 2x^2$

$$A'(x) = 80 - 4x$$

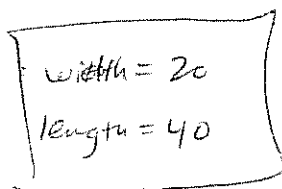
$$0 = 80 - 4x$$

$$x = 20$$



$x = 20$  is a max

$\Rightarrow$  dimensions



4. (2 points) The temperature of a cup of coffee  $t$  minutes after it is poured is given by the equation

$$T = 70 + 100e^{-0.0446t}$$

where  $T$  is degrees Fahrenheit. When will the coffee be cool enough to drink (say, 120 degrees)? Round answer to the nearest minute.

$$120 = 70 + 100e^{-0.0446t}$$

$$50 = 100e^{-0.0446t}$$

$$\frac{50}{100} = e^{-0.0446t} \quad (\text{take } \ln \text{ of both sides})$$

$$\ln\left(\frac{1}{2}\right) = -0.0446t$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.0446} = 15.54 \approx 16 \text{ minutes}$$